RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, AUGUST 2021

SECOND YEAR (BATCH 2019-22)

Date : 09/08/2021 **PHYSICS (Honours)**

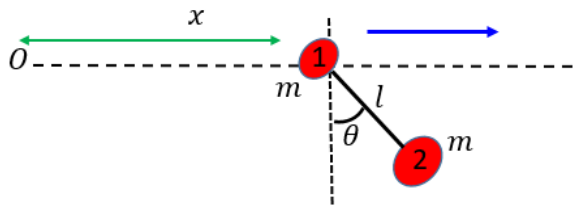
Time : 11.00 am – 1.00 pm **Paper : IX [CC 9]** Full Marks : 50

UNIT - I

Answer **any three** questions [3×10]

1. a) Using the Lagrangian equations of motion solve for the motion of a simple pendulum and determine its time period. [5]

   b) Consider a system of particles with the same mass m, shown in the figure below. The entire system is moving towards the right. Also particle 2 oscillates like a pendulum with its fixed point at particle 1. Determine the Lagrangian of the system and hence solve for its equation of motion. [5]



2. a) Compute the degrees of freedom for the following systems

   (i) Particle moving on the surface of a sphere.

   (ii) Five particles moving on a plane.

   (iii) Dumb bell moving in 3D space.

   (iv) Seven particles moving in 3D space such that the distance between any pair is always fixed.

   (v) A pendulum oscillating about a fixed point, such that the fixed point itself is moving along the x-axis. [5]

   b) Consider a particle moving under the influence of a force that depends only on its distance from the origin. Write down the Lagrangian. Just by looking at the Lagrangian (i.e. without solving for the equations of motion), determine the conserved quantity. Explain why? [5]

3. a) Consider a particle of mass m falling vertically downwards. The distance z it travels in time t is given by $z = at + bt^2$. Also, it is given that it travels a distance z_0 in time $t_0 = \sqrt{\frac{2z_0}{g}}$. Write down the Lagrangian for the system. Show that the action integral $\int_0^{t_0} L\, dt$ is an extremum only if $a = 0$ and $= \frac{g}{2}$. [5]

   b) Consider a particle moving on a surface of a cylinder of radius a. If the force acting on the particle is $F(r) = -kr$, determine the Lagrangian for the particle. Write down the Hamiltonian from the Lagrangian and hence its equation of motion. [5]

4. a) Consider two identical Harmonic oscillators which are coupled together. Their Kinetic and potential energies are $T = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2)$ and $V = \frac{1}{2}k(q_1^2 + q_2^2) - hq_1q_2$. Write down the Lagrangian and hence the Hamiltonian for the system. Determine the frequencies of the normal modes. [5]

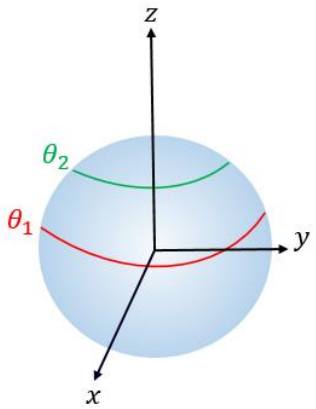
b) The potential energy of a particle is

$V(x) = 3x^4 - 8x^3 - 6x^2 + 24x.$

Determine the points of stable and unstable equilibrium. [5]

5. a) Consider a symmetrical top with $I_1 = I_2 \neq I_3$. Determine its kinetic energy of rotation with respect to the principal axis in terms of the Euler's angles. [5]

b) Describe the motion of a heavy symmetric top, by sketching the locus of the body z' axis on the surface of a unit sphere



For the following cases : $\dot{\phi} > 0, \dot{\phi} < 0, \dot{\phi} = 0$ at intermediate value of θ_1 and θ_2 and $\dot{\phi} = 0$ at $\theta = \theta_1$. Denote the direction of the paths by arrows and explain your answer in each case. [5]

Given:

$$\dot{\phi} = \frac{b - a\cos\theta}{\sin^2\theta}$$

UNIT - II

Answer **any two** questions [2×10]

6. a) A pilot is supposed to fly due east from A to B and back again to A due west. The velocity of the plane in air is *u'* and the velocity of the air with respect to the ground is *v*. The distance between A and B is L and the plane's speed in air u' is constant. Determine the time for a round trip if (i) *v=0* (still air), (ii) the air velocity is due east (or west), (iii) the air velocity is due north (or south). (iv) In part (ii) and (iii) we must assume that *v<u'*. Why? (v) Draw an analogy to the Michelson-Morley experiment. [1+1+1+1+1]

b) (i) How Lorentz-Fitgerald contraction hypothesis explains the null result of Michelson-Morley experiment? How this hypothesis was abandoned? [1+1]

(ii) How Ether-Drag hypothesis explains the null result of Michelson-Morley experiment? How this hypothesis was abandoned? [1+1]

(iii) How the postulates of Special Relativity explain the null result of Michelson-Morley experiment? [1]

7. a) Show that the following operator is invariant under Lorentz transformation: [5]

$$= \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

b) The most distant star in our Galaxy is approximately at 100,000 light years away. Explain in terms of time dilation (or length contraction) how it is possible, in principle, for a human being to reach this star within his normal life span. Estimate what uniform velocity he would require to get there in 10 years (measured in the inertial frame in which he is at rest)? [1+1]

c) Suppose that cosmic rays produce 10,000 muons at a height of 50 km above sea level. If their proper mean life is 2.2 μs and speed is 0.995 c, find how many of them reach the sea level without decay. [2]

d) A spacecraft passes Saturn with a speed of 0.9c relative to Saturn. A second spacecraft is observed to pass the first one (going in the same direction) at relative speed of 0.2c. What is the speed of the second spacecraft relative to Saturn? [1]

8. a) How can you synchronize two clocks located at different locations? [2]

b) Explain using space-time diagram if an event A precedes an event B at the same point in one frame of reference, will A precede B in all other inertial reference frame? Will they occur at the same point in any other inertial frame? Will the time interval between the events be the same in any other inertial frame? [1+1+1]

c) An event occurs at the origin of an inertial frame S at $t=0$, Another event occurs at $x=4c$, $y=0$, $z=0$ at $t=5$ s relative to S. What is the space-time interval between these events? Determine the velocity of the inertial frame S' in which the two events are recorded at same point in space. What is the time interval between these events at S'? [1+1+1]

d) Calculate the Doppler shift in wavelength (using relativistic formula) for light of wavelength 6000 Å (i) when the source approaches the observer at a velocity 0.2c and (ii) when the source moves transversely to the line of sight at a velocity 0.6 c. [1+1]

9. a) An unstable neutral particle decays into two charged particles of kinetic energies 190 MeV and 30 MeV and momenta 300 MeV/c and 240 MeV/c respectively. Determine the masses of the decay products. If the angle between the decay particles is 45°, determine the rest mass, momentum and kinetic energy of the neutral particle. [2+1+1+1]

b) How the four components of force transform under Lorentz Transformation? [5]

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